1998 JRAHS 3U TRIAL.

QUESTION 1:

(a) Sketch the graph of $y = 2\cos^{-1}\left(\frac{x}{3}\right)$.

(b) If $p = \log_a 3$ and $q = \log_a 2$, express $\log_a(4\frac{1}{2})$ in terms of p and q.

(c) If $\int_{-2}^{2\sqrt{3}} \frac{dx}{\sqrt{16 - x^2}} = k\pi$, find the value of k.

(d) The line $2x + y - 10 = 0$ is tangent to a circle with centre $(-3,1)$. Find the radius of the circle and hence write down the equation of the circle.(Do not expand your answer)

QUESTION 2: (START A NEW PAGE)

(a) (i) If $\sin^2\theta = A + B\cos 2\theta$ write down the values of A and B.

(ii) Hence or otherwise evaluate $\int_0^{\frac{\pi}{8}} \sin^2 3x \, dx$.

(b) Find the value of the constant term in the expansion of $\left(3x^2 + \frac{2}{x}\right)^{12}$.

(c) (i) Sketch $y = \frac{x}{x^2-2}$ clearly showing all intercepts with the co-ordinate axes and all asymptotes.

(ii) Hence, or otherwise, solve $\frac{x}{x^2-2} \geq 1$.

QUESTION 3: (START A NEW PAGE)

(a) The motion of an object is defined by the equation $x = 3\sin\pi t$. Write down the amplitude and period of the motion.

(b) (i) Show that $\frac{d}{d\theta}\left(ln(\sec\theta + \tan\theta)\right) = \sec\theta$.

(ii) Use the substitution $x = \cos\theta$ to evaluate $\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x} \, dx$.

(c) Use Mathematical Induction to prove that:

$$\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1} \text{ for integers } n \geq 1$$

QUESTION 4: (START A NEW PAGE)

(a) (i) Find $\frac{d}{dx}(x\tan x)$.

(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx$.

(b) Five letters are chosen at random from the word FLAVOURS and arranged in a line.

(i) How many different five letter "words" can be formed?

(ii) Find the probability that a five letter "word" formed from the letters of FLAVOURS will have at least one vowel.

(c) A 5 metre ladder rests with one end against a vertical wall and the other end on horizontal ground which is level with the base of the wall. The end which is in contact with the ground slips away from the wall at a constant rate of 0.1 m/s. Find the rate (in radians/sec) at which the angle between the ladder and the ground is decreasing when the end of the ladder is 3 metres from the wall.

QUESTION 5: (START A NEW PAGE)

$P(4p,2p^2)$ is a point on the parabola $x^2 = 8y$ with focus S.

i) Prove that the equation of the tangent at P is given by $y = px - 2p^2$.

ii) Show that the equation of the line through S and perpendicular to SP is
$2px + (p^2 - 1)y = 2(p^2 - 1)$.

iii) The tangent and this line meet at M. Prove that the coordinates of M are $\left(\frac{2(p^2 - 1)}{p}, -2\right)$.

iv) Prove that the area of $\Delta PSM = \frac{2(p^2+1)^2}{|p|}$.

v) Find the value(s) of p so that the area of ΔPSM is a minimum.

QUESTION 6: (START A NEW PAGE)

(a) (i) Write down the expansion of $(a + b)^7$.

(ii) If there is a 2[illegible]% chance that it will rain on any day, find the probability that it will rain on at most 2 days of a seven day athletics carnival. (Give your answer to the nearest percent)

(b) In a colony of bees it is found that the number (N) infected by a virus at any time t, in months, is given by

$$N = \frac{600}{1 + Ae^{-0.05t}}$$

(i) If initially there are 50 infected bees find the value of A.

(ii) Find the time taken before there are 100 infected bees. (Give your answer to the nearest month)

(iii) Show that $\frac{dN}{dt} = \frac{N([illegible]50 - N)}{[illegible]3000}$.

(iv) Find the rate at which the infection is spreading when there are 120 infected bees

QUESTION 7: (START A NEW PAGE)

(a) i) Show that $\ddot{x} = v\frac{dv}{dx}$

ii) An object moves so that its velocity (v) at position (x) is given by $v = \frac{[illegible]}{2x - 3}$.

Show that $\ddot{x} = -\frac{v^2}{2}$.

(b) A missile is fired from ground level into the air at a velocity of 40 ms^{-1} and at an angle α with the horizontal. A short time later another missile is fired from the same point and with the same speed but at a different angle β. Both missiles hit the same target at the same time. The target is 55m above the ground and 80 m from the point of firing. Take $g=10\ ms^{-2}$ and neglect air resistance.

(i) Write down expressions for the horizontal and vertical components of the position of the first missile t seconds after it is fired.

(ii) Show that the path of the first missile is given by $y = x\tan\alpha - x^2\left(\frac{\sec^2\alpha}{320}\right)$.

(iii) Find values of $\tan\alpha$ and the value of $\tan\beta$.

(iv) Determine the time difference between the firing times of the two missiles.

QUESTION 1

(a) —

(b) $2q - p$

(c) $k = \frac{1}{2}$

(d) $r = 3\sqrt{5}$

$(x+3)^2 + (y-1)^2 = 45$

QUESTION 2

(a) (i) $A = \frac{1}{2}$ $B = -\frac{1}{2}$

(ii) $\frac{1}{4}\left(\frac{\pi}{4} - \frac{1}{3\sqrt{2}}\right)$

(b) ${}^{12}C_8 \cdot 3^4 \cdot 2^8$

(c) (i) —

(ii) $-\sqrt{2} < x \leq -1$, $\sqrt{2} < x \leq 2$

QUESTION 3

(a) amp $= 3$

period $= 2$

(b) (i) —

(ii) $\ln(2+\sqrt{3}) - \sqrt{3}/2$

(c) —

QUESTION 4

(a) (i) $x\sec^2 x + \tan x$

(ii) $\frac{\pi}{4} + \ln\left(\frac{1}{\sqrt{2}}\right)$

(b) (i) 6720

(ii) 55/56

(c) 0.025 rad/sec

QUESTION 5

(i) —

(ii) —

(iii) —

(iv) —

(v) $p = \pm 1/\sqrt{3}$

QUESTION 6

(a) (i) $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$

(ii) $\doteqdot 85\%$

(b) (i) $A = 8$

(ii) $\doteqdot 28$ mths.

(iii) —

(iv) $\doteqdot 1.2$ bees/mth.

QUESTION 7

(a) (i) —

(ii) —

(b) (i) $x = 40\cos\alpha \cdot t$

$y = -5t^2 + 40\sin\alpha\, t$

(ii) $\tan\alpha = 5/2$

$\tan\beta = 3/2$

(iv) diff $= \sqrt{29} - \sqrt{13}$ sec